Le Congrès des Jeunes Chercheurs en Mathématiques et Applications 2023

CJC-MA 2023

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For more details, check the conference website.

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Plenary talk – Monday 13:30-14:30 – Théâtre e.070

Dérivation d'un modèle d'écoulement compressible avec bulles

Hélène Mathis

On s'intéresse à la dérivation d'un modèle d'écoulement compressible constitué d'un fluide et de bulle, en utilisant une méthode d'homogénéisation.
À l'échelle microscopique, le fluide est décrit par le système de Navier-Stokes compressible, tandis que le comportement des bulles est décrit par les lois de Newton. Une originalité du modèle réside dans la prise en compte de la tension de surface aux interfaces fluide-bulles. En considérant que le nombre de bulles diverge, on propose la construction de quantités macroscopiques, comme le taux de présence du fluide ou sa masse volumique. On obtient un système d'équations associées à ces quantités qui est constitué d'une loi d'évolution de la fraction de volume et, plus original, d'une équation cinétique satisfaite par la fonction de distribution des bulles.
Il s'agit d'un travail réalisé avec M. Hillairet et N. Seguin.

Monday 14:45-15:45

Statistics – e.068

Reference prior construction for Bayesian inference applied to seismic fragility curve estimation

Antoine Van Biesbroeck

Seismic fragility curves of mechanical structures are key quantities of interest for probabilistic seismic risk assessment studies. They express the probability of failure of the mechanical structure conditional to a scalar value derived from the seismic ground motions, coined intensity measure (IM). Evaluation of these curves can be carried out by using Monte Carlo methods and mechanical numerical simulations with artificial seismic signals. Nevertheless, when resorting to a complex and detailed modeling, the number of data available is limited due to the calculation burden, making estimation of those curves challenging.

The Bayesian viewpoint makes it possible to learn more efficiently the parameters which determine the fragility curves. It avoids the generation of unrealistic samples – which is common with classical methods – and which can lead to degenerate curves such as unit step functions for example. With a small dataset, it is known that the choice of prior has a significant influence on the posterior distribution. In practice, however, its definition requires information that is not necessarily available or difficult to justify, so it may be debatable.

Reference prior theory provides answers to this problem. It relies on the maximization of an objective criterion that is based on a mutual information metric. To support the prior choice, we propose an enrichment of this criterion with a wider range of mutual information measures which we prove to be asymptotically maximized by the Jeffreys' prior. We then suggest an implementation of this prior in the context of seismic fragility curve estimation. The posterior distributions of the parameter of interest are compared both theoretically and numerically with the ones obtained with classical priors from the literature.

While our results show the robustness of Jeffreys' prior which outperforms the ones of the literature, they highlight the sensitivity of the estimates with respect to the chosen prior. Also, by investigating different IMs (Peak Ground Acceleration vs. Pseudo Spectral Acceleration), we unveil degenerative phenomena that may happen when using an IM that is correlated to the structure response and may be difficult to handle by any non-informative prior.

Our conclusion emphasizes the importance of an appropriate and objective construction for the prior in Bayesian studies.

Estimation of a pure-jump stable Cox-Ingersoll-Ross process

Elise Bayraktar

We consider a pure-jump stable Cox-Ingersoll-Ross (α-stable CIR) process driven by a non-symmetric stable Lévy process with jump activity $\alpha \in (1, 2)$ and we address the joint estimation of drift, scaling and jump activity parameters from high-frequency observations of the process on a fixed time period. We first prove the existence of a consistent, rate optimal and asymptotically conditionally gaussian estimator based on an approximation of the likelihood function. Moreover, uniqueness of the drift estimators is established assuming that the scaling coefficient and the jump activity are known or consistently estimated. Next we propose easy-to-implement preliminary estimators of all parameters and we improve them by a one-step procedure.

Inverse problems – Théâtre e.070

Dictionary-based Model Reduction for State Estimation

Alexandre Pasco

We consider the problem of state estimation from **m** linear measurements, where the state **u** to recover is an element of the manifold **M** of solutions of a parameter-dependent equation. The state is estimated using a prior knowledge on **M** coming from model order reduction. Variational approaches based on linear approximation of **M**, such as (3), yields a recovery error limited by the Kolmogorov **m**-width of **M**. To overcome this issue, piecewise-affine approximations (2) of the manifold have also be considered, that consist in using a library of linear spaces among which one is selected by minimizing some distance to **M**. In this paper, we propose a state estimation method relying on dictionary-based model reduction, where a space is selected from a library generated by a dictionary of snapshots, using a distance to the manifold. The selection is performed among a set of candidate spaces obtained from the path of a **l1**-regularized least-squares problem. Then, in the framework of parameter-dependent operator equations (or PDEs) with affine parameterizations, we provide an efficient offline-online decomposition based on randomized linear algebra (1), that ensures efficient and stable computations while preserving theoretical guarantees.

References :

(1) Balabanov, O., Nouy, A. : Randomized linear algebra for model reduction-part II: Minimal residual methods and dictionary-based approximation. https://doi.org/10.1007/s10444-020-09836-5
(2) Cohen, A., Dahmen, W., Mula, O., Nichols, J.: Nonlinear Reduced Models for State and Parameter Estimation. https://doi.org/10.1137/20M1380818
(3) Maday, Y., Patera, A.T., Penn, J.D., Yano, M.: A parameterized-background data-weak approach to variational data assimilation: Formulation, analysis, and application to acoustics. https://doi.org/10.1002/nme.4747

The linear sampling method for random sources

Hadrien Montanelli

We present in this talk an extension of the linear sampling method for solving the sound-soft inverse acoustic scattering problem with randomly distributed point sources. The theoretical justification of our method is based on the Helmholtz–Kirchhoff identity, the cross-correlation between measurements, and the volume and imaginary near-field operators. Implementations in MATLAB using boundary elements, the SVD, Tikhonov regularization, and Morozov's discrepancy principle will also be discussed.

Theoretical analysis of PDEs – sc.071

Cahn-Hilliard system: From nonlocal to local

Charles Elbar

Joint work with José Carrillo (Oxford) and Jakub Skrzeczkowski (Warsaw). We provide a rigorous mathematical framework to establish the limit of the nonlocal model of cell-cell adhesion introduced in arXiv:2206.14461 to a local model. When the parameter of the nonlocality goes to 0, the system tends to a Cahn-Hilliard system with degenerate mobility and cross-interaction forces. The proof is based on the strategy developed in arXiv:2208.08955 for the single Cahn-Hilliard equation. Numerical simulations show that the latter model preserves the diversity of cell sorting patterns seen in experiments and previous nonlocal models. It also has the advantage of having explicit stationary solutions.

Coupling kinetic and fluid equations: the thick sprays regime, theory and numerics

Victor Fournet

In this talk, we are interested in a coupled system between kinetic and fluid equations, describing particles evolving in an underlying gas. In particular, we talk about the "thick sprays" regime where the volume fraction of the particles in not negligible. I will review recent progress of thick sprays equations, and present some numerical result. In particular I will present an interesting analogy between thick sprays and Vlasov-Poisson equations, and show through numerics and theory that the thick sprays equations presents a behavior similar to the so-called Landau damping in the linear regime.

Probability – sd.206

Monday 14:45-15:45

Régularité de la constante de temps pour le modèle de percolation de dernier passage sur les graphes complets acycliques orientés

Benjamin Terlat

Le modèle est défini de la manière suivante : On considère un graphe complet à n sommets numérotés de 1 à n dans lequel toutes les arêtes sont orientées dans le sens de la numérotation (des petits vers les grands sommets). Les arêtes sont munies de poids aléatoires à valeurs réelles, que l'on suppose i.i.d. On considère que le poids d'un chemin orienté est donné par la somme des poids de ses arêtes. La quantité étudiée est le maximum des poids des chemins orientés partant de 1 et finissant en n. Par sous-additivité, ce poids maximal est équivalent à une constante fois n, le nombre de sommets dans le graphe, lorsque n tend vers l'infini. Cette constante est appelée constante de temps et ne dépend que de la loi des poids des arêtes. Dans le cas où les poids des arêtes valent 1 avec probabilité p et $-\infty$ avec probabilité $(1-p)$, Mallein et Ramassamy ont montré l'analyticité de la constante de temps en p. Dans cet exposé, nous montrons que ce résultat s'étend aux mesures supportées par un nombre fini N d'atomes. Dans le cas où $N = 2$ et où les atomes sont positifs, la constante de temps est une fonction rationnelle des atomes et de leurs probabilités. Dans le cas général où le support de la loi des poids est majoré, la constante de temps est strictement croissante pour l'ordre stochastique et continue en la loi des poids des arêtes. (Exposé *basé sur arXiv:*2303.11927)

Zero noise limit for ODE with fractional noise

Łukasz Mądry

A classical manifestation of regularization by noise is that adding an irregular term to an ill-posed equation may restore well-posedness (existence/uniqueness). A natural question is then, in the limit where the coefficient in front of the noise is taken to zero, whether this selects one (or several) particular solutions to the original equation (this is typically referred to as "selection by noise"). In the case of one-dimensional ODEs, perturbed by a Brownian motion, Bafico and Baldi '82 showed that this procedure selects extremal solutions, i.e. those that exit the problematic point instantly. We extend this result to the case of fractional noise (and obtain in addition some exponential concentration estimates). The main difficulty lies in the absence of the Markov property for the system. Our proof is based on the dynamical approach of Delarue-Flandoli '14, combined with recent progress in regularisation by fractional noise (Catellier-Gubinelli '16), and techniques coming from the study of ergodicity of fractional SDEs (Hairer '05, Panloup-Richard '20). Based on joint work with Paul Gassiat (Univ. Paris-Dauphine).

Monday 16:15-17:45

Theoretical and numerical analysis of PDEs – e.068

Limite continue de l'équation de Schrödinger discrète

Quentin Chauleur

On s'intéresse dans cet exposé à la limite de l'équation de Schrödinger discrète sur le réseau $h\{Z\}\hat{}d$ pour $d = 1, 2$ quand la taille de la grille h tend vers 0. En particulier, on cherche à obtenir des taux de convergence explicites dans des espaces de Sobolev arbitraires de la solution discrète vers celle de l'équation continue pour tout temps. La démonstration repose sur le contrôle de l'évolution des normes de Sobolev discrètes de la solution, ainsi que sur des estimations bilinéaires de l'interpolation de Shannon. L'estimation obtenue est dite compatible, au sens où plus de régularité sur la condition initiale permet d'obtenir une meilleure convergence en h.

Observability of the Schrödinger equation with confining potential

Antoine Prouff

We consider a quantum particle evolving in the Euclidean space and trapped in a confining potential. Its wave function (whose modulus squared corresponds to a probability density) solves the Schrödinger equation. The observability problem consists in wondering which are the open sets of the Euclidean space in which one can see the particle over a time interval $(0,\ T)$ with a positive probability, independent of the intial state of the particle. Observability is true if one observes from the whole space. That being said, it is interesting for applications to understand which are the smallest possible observation sets. We shall give a characterization of these open sets where observability holds (up to slightly enlarging the observation set) in terms of the trajectories of the underlying classical mechanics. We will illustrate this result with an example involving harmonic oscillators.

Lebesgue spaces with variable exponent: some applications to the Navier-Stokes equations

Gaston Vergara Hermosilla

In this article we study some problems related to the incompressible 3D Navier-Stokes equations from the point of view of Lebesgue spaces of variable exponent. These functional spaces present some particularities that make them quite different from the usual Lebesgue spaces. Indeed, some of the most classical tools in analysis are not available in this framework and we will give some ideas to overcome some of the difficulties that arise in this context.

Mathematics in health and biology – sc.071

Activation processes of flagellated micro-swimmers

Irene Anello

Swimming in a fluid at microscopic scale is at the heart of many questions pertaining to biology, soft matter physics and micro-robotics. It usually concerns a various range of viewpoints, from macroscopic one, such as hydrodynamics and elasticity, to microscopic ones, such as internal activity at molecular scale. Motivated by this, we study the activation processes of flagellated micro-swimmers investigating microscopic details inside the flagellum. The flagellum is composed of a structure called axoneme, constitute of nine filament pairs along which are disposed force-generating elements called molecular motors. After describing the biology behind it, we first model the interaction between motors and filaments individually before introducing a mathematical representation of the whole nine filaments system. The aim is to couple this complex microscopic description with a macroscopic beam equation for flagellated swimmers to observe oscillations and collect various data, such as the critical oscillation frequency.

Generalized reducible SIS model

Kacem Lefki

In this talk, we present a non-homogeneous deterministic SIS (Susceptible - Infected - Susceptible) model, with a possibly infinite set of features. The behaviour of the system is well-known when the underlying operator is irreducible (informally: every individual may infect every other individual)
We characterize the the existence of an endemic equilibrium using the basic reporduction number R0, and prove the convergence of the solutions to an endemic equilibrium when R0 $>$ 1, without any assumption of irreducibility.
We will present the link between these results and the theory of positive operators in Banach lattices, especially the notion of invariant set.

Modèles de nageurs bi-directionnels à bas nombre de Reynolds

Jessie Levillain

De nombreuses modélisations simples tentent d'expliquer la locomotion de micro-organismes qui évoluent dans un fluide. En effet, la compréhension de ces systèmes est cruciale pour de nombreux domaines des sciences, et en particulier en biologie, car des organismes tels que les bactéries, les spermatozoïdes ou les micro-algues sont à l'origine de la vie. En particulier, le célèbre théorème de la coquille Saint-Jacques de Purcell impose des conditions sur la structure du nageur et son nombre de degrés de liberté.
Il a récémment été montré qu'il était possible de modéliser des nageurs à bas Reynolds possédant seulement un degré de liberté, mais qui pouvaient malgré tout passer au-dessus du théorème de la coquille Saint-Jacques. Un exemple parmi tant d'autres est étudié par Montino et De Simone, qui modélisent un nageur à trois sphères avec un seul bras actif et un ressort passif, basé sur le nageur avec deux bras de Najafi et Golestanian.
Dans cette présentation, nous généralisons ce modèle en augmentant le nombre de ressorts, avant de montrer que la direction dans laquelle nage le modèle peut être inversée grâce à un jeu sur la fréquence du bras actif.

Numerical methods for PDEs – sd.206

A fully adaptive explicit stabilized integrator for advection–diffusion–reaction problems

Ibrahim Almuslimani

We introduce a novel second order family of explicit stabilized Runge–Kutta–Chebyshev methods for advection–diffusion–reaction equations. The new methods outperform existing schemes for relatively high Peclet number due to their favorable stability properties and explicitly available coefficients. The construction of the new schemes is based on stabilization using second kind Chebyshev polynomials first used in the construction of the stochastic integrator SK-ROCK. An adaptive algorithm to implement the new scheme is proposed. This algorithm is able to automatically select the suitable step size, number of stages, and damping parameter according to the value of the Peclet number at each integration step. We present numerical experiments that illustrate the efficiency of the new algorithm.

Analyse discrète en temps pour l'algorithme de décomposition de domaine OSWR

Arthur Arnoult

Les simulations numériques engendrent souvent des problèmes couteux (par exemple avec un maillage fin), dont la résolution numérique peut être très longue.
La décomposition de domaine est une stratégie de résolution des EDPs qui a pour objectif de répondre à une telle problématique.
Le principe est de décomposer le domaine physique en un certain nombre de sous-domaines. Ensuite, l'équation initiale est résolue sur chaque sous-domaine un certain nombre de fois, successivement, en échangeant à chaque itération des informations astucieusement choisies entre les sous domaines. L'objectif est alors de choisir le meilleur opérateur de permettant de transférer les informations à l'interface afin de minimiser le nombre d'itérations de décomposition de domaine à effectuer (voir, par exemple, [?], [?]).
Pour les problèmes en temps, l'algorithme OSWR (Optimized Schwarz waveform relaxation) désigne l'algorithme où cet opérateur est de type Robin, i.e. sur deux sous domaines adjacents dont la frontière est de normale $\mathbf{n}$:

$$\mathcal{B} = \alpha + \partial_{\mathbf{n}} \tag{1}$$

Cet opérateur fait intervenir un paramètre α qui est libre et à choisir afin d'optimiser la convergence de l'algorithme.
Dans cet exposé, nous prenons l'exemple de l'équation de la chaleur en une dimension d'espace :

$$\mathcal{L}u = \partial_t u - \nu \partial_{xx} u = f \tag{2}$$

Après avoir introduit une partition du domaine Ω en deux sous domaines Ω_1 et Ω_2, l'algorithme s'écrit

$$\begin{array}{rlllrll} \mathcal{L}u_1^\ell & = f & \text{dans } \Omega_1 \times (0,T) & & \mathcal{L}u_2^\ell & = f & \text{dans } \Omega_2 \times (0,T) \\ u_1^\ell(\cdot,0) & = u_0 & \text{dans } \Omega_1 & & u_2^\ell(\cdot,0) & = u_0 & \text{dans } \Omega_2 \\ \mathcal{B}u_1^\ell & = \mathcal{B}u_2^{\ell-1} & \text{sur } \{0\} \times (0,T) & & \mathcal{B}u_2^\ell & = \mathcal{B}u_1^{\ell-1} & \text{sur } \{0\} \times (0,T) \\ \lim\limits_{x\to-\infty} u_1^\ell(x,\cdot) & & \text{borné sur } (0,T) & & \lim\limits_{x\to+\infty} u_2^\ell(x,\cdot) & & \text{borné sur } (0,T) \end{array} \tag{3}$$

Nous avons ainsi défini deux suites $(u_i^\ell)_\ell$, $i = 1, 2$, dont on va montrer qu'elles convergent vers $u|_{\Omega_i}$. La problématique est alors de choisir le paramètre libre α afin d'accélérer la convergence de l'algorithme. L'étude (publiée dans [?]) propose une nouvelle manière d'analyser l'algorithme, reposant sur l'introduction d'un schéma numérique en temps.
L'étude met en évidence que le meilleur paramètre α dépend du nombre d'itérations de décomposition de domaine qui est visé. Cette analyse apporte de nouveaux résultats de convergence et permet la détermination de paramètres approchant le meilleur paramètre α, en fonction du nombre d'itérations de décomposition de domaine visé. Nous présenterons des résultats numériques illustrant l'efficacité des paramètres ainsi définis.

Conditions de bord continues à l'interface de deux fluides.

Francois Legeais

On s'intéresse à l'étude de deux systèmes de stokes stationnaires pour deux fluides distincts reliés par une condition de continuité à l'interface. On verra que les solutions de ce système peuvent s'obtenir comme solutions limites d'un système similaire avec conditions de Robin (mixtes) à l'interface.
Les conditions de Robin permettent une modélisation numérique grâce à un algorithme de type Schwartz (ce que ne permet pas la condition de bord continue) sur Freefem++.
Nous verrons en conclusion et en perspective un système couplé de deux fluides plus complexe de type navier-stokes stationnaires avec viscosité de Smagorinsky et conditions de frictions au bord. L'idée étant la même que pour la première étude avec à la clé simulations numériques de type Schwartz.

Monday from 17:45 – Espace capable h.075

Session posters

K. Baalu, A. Barry, Aurélie Bigot, C. Cardoen, R. Carpintero Perez, F. Chaaban, G. Chennetier, G. Claret, S. Claret, M.Crespo, A. Drouard, I. Hasenohr, E. Jaber, A. Kabalan, G. Leloup, P. Mackowiak, R. Mottier, A. Parigaux, M. Payan, S. Ruget, A. Safa, A. Sultanov, M.Thorez

Plenary talk – Tuesday 9:30-10:30 – Théâtre e.070

Percolation et géométrie algébrique

Damien Gayet

En percolation planaire, on enlève des arêtes de $\mathbb{Z}^2$ aléatoirement et indépendamment les unes des autres, et on tente de comprendre la statistique de la géométrie du sous-graphe résultant. En géométrie algébrique aléatoire planaire, on prend au hasard les coefficients d'un polynôme réel en deux variables et on tente de comprendre la statistique de la géométrie de son lieu d'annulation. À part la similarité des buts, on ne voit pas bien comment ces deux objets peuvent se parler. Nous verrons que de façon surprenante, c'est bien le cas.

Tuesday 11:00-12:30

Theoretical and numerical analysis of PDEs – e.068

Green's function pointwise estimates for spectrally stable stationary discrete shock profiles

Lucas Coeuret

The aim is to present a result on the stability of discrete shock profiles for one-dimensional systems of conservation laws. It provides precise estimates of the Green's function associated with stationary discrete shock profiles of finite difference schemes with numerical viscosity. This result is the first step towards a result of nonlinear orbital stability for such discrete shock profiles.

Grid-free Weighted Particle method applied to the Vlasov-Poisson equation

Yoann Le Henaff

Le but de cet exposé est de présenter une méthode numérique d'approximation pour les solutions de l'équation de Vlasov-Poisson sans terme source. Celle-ci se base sur une représentation exacte d'un champ electrique approché.
Il s'agit d'une méthode particulaire qui ne présente pas les inconvénients habituels des méthodes de type Particle-In-Cell car on évite l'étape de déposition sur une grille qui introduit du bruit statistique. La convergence de cette méthode sera présentée, ainsi que les aspects numériques liés à cette méthode.
L'exposé sera essentiellement tiré du papier suivant : https://hal.science/hal-03736227.

On the generalized hyperbolicity of the hydrostatic Euler equations

Chourouk El Hassanieh

A priori, the free-surface hydrostatic Euler system does not fall into any classical classification of PDEs, including hyperbolic systems. However, the work introduced in (3, 5) demonstrates the possibility of establishing an analogy. This is achieved throughthe use of a transformation that maps the time-dependent domain onto a fixed domain, where the variables are expressed in semi-Lagrangian coordinates. Unlike the classical σ*-transformation*, which solely addresses the geometry of the occupied domain, this transformation incorporates an additional constraint. Consequently, the system can be rewritten in a generalized quasi-linear form with a matrix operator. Provided some regularity assumptions, we prove the existence and uniqueness of this transformation and consequently the equivalence between the two systems. We extend the works in (2, 3, 4) by giving a full decomposition of the spectrum of the matrix operator. We show that the system admits two real and distinct eigenvalues under a certain Hölder-regularity of the variables below which the existence of these two eigenvalues is no longer valid. We address the notion of Riemann invariants which differs from the classical notion for hyperbolic conservation laws. We note that these classical notions remain not very well understood in the context of the generalized hyperbolic system under study however they enable us to make a link between the classical hyperbolic theory and the generalized quasi-linear system. This work is complemented by a vertical discretization of the fluid domain (1) where we study the hyperbolicity of the discretized system using classical tools.

(1) Emmanuel Audusse, Marie-Odile Bristeau, Benoît Perthame, and Jacques Sainte-Marie. A multilayer Saint-Venant system with mass exchanges for shallow waterflows. Derivation and numerical validation.ESAIM, Math. Model. Numer. Anal.,45(1):169–200, 2011.
(2) A. A. Chesnokov, G. A. El, Sergey L. Gavrilyuk, and M. V. Pavlov. Stability of shear shallow water flows with free surface.SIAM J. Appl. Math., 77(3):1068–1087, 2017.
(3) V. M. Teshukov. On hyperbolicity of long-wave equations.Sov. Math., Dokl., 32:469–473, 1985.
(4) Vladimir Teshukov, Giovanni Russo, and Alexander Chesnokov. Analytical and nu-merical solutions of the shallow water equations for 2d rotational flows.Math. ModelsMethods Appl. Sci., 14(10):1451–1479, 2004.
(5) V. E. Zakharov. Benney equations and quasiclassical approximation in the method ofthe inverse problem.Funct. Anal. Appl., 14:89–98, 1980.

Probability – sc.071

Caractérisation des extenions compactes confinées

Séverin Benzoni

Le sujet de cet exposé est l'étude des systèmes dynamiques mesurés, c'est-à-dire un triplet (X, μ, T) tel que (X, μ)est un espace de probabilité et T est une transformation inversible qui préserve μ. Plus précisément, nous nous intéressons aux propriétés des extensions de systèmes dynamiques. La notion principale introduite dans l'exposé est celle d'extension confinée, basée sur des propriétés de couplage. Nous montrons l'interêt de cette notion en énonçant certains résultats de relèvement associés. Enfin, nous aborderons la question de la caractérisation du confinement dans la famille des extensions compactes, puis nous donnerons une application de cette caractérisation sur un exemple.

Sur la croissance démographique avec les catastrophes

Branda Goncalves

Dans cet exposé, nous parlerons d'une classe particulière de processus de Markov déterministes par morceaux (PDMP) qui sont des versions catastrophes semi-stochastiques de modèles de croissance démographique déterministes. Entre les sauts successifs, le processus suit un flux décrivant la croissance déterministe de la population. De plus, aux moments aléatoires des sauts, régis par des taux dépendant de l'état, la taille de la population diminue d'une quantité aléatoire de sa taille actuelle, un événement pouvant conduire à une extinction locale (ou totale) instantanée. Sans cet exposé, je discuterai des conditions sous lesquelles de tels processus sont récurrents (positifs ou nuls) ou transitoires. Si le temps le permet, la question de la finitude du temps jusqu'à l'extinction sera abordée ainsi que l'évaluation du temps moyen jusqu'à l'extinction lorsque le dernier est fini.

Analyse de convergence de CMA-ES

Armand Gissler

En optimisation "boîte noire", les stratégies d'évolution (ES) forment une classe d'algorithmes stochastiques qui ne font pas appel aux dérivées de la fonction f à minimiser (ou maximiser). À la place, seulement le classement des valeurs par f de différents points candidats sont utilisées. Les stratégies d'évolution avec adaptation de la matrice de covariance (CMA-ES) font non seulement une approximation du minimum de f par une distribution normale, mais aussi de l'inverse de sa Hessienne, toujours sans calculer les dérivées de la fonction. Je discuterai d'une analyse de convergence de CMA-ES qui repose sur la stabilité d'une chaîne de Markov sous-jacente. Il s'agit d'une approche qui a pu démontré la convergence d'autres algorithmes ES, et j'expliquerai comment l'appliquer à CMA-ES.

Riemannian analysis and geometry – sd.206

Metamorphoses of manifold-valued images

Guillaume Sérieys

Manifold-valued images are of interest in medical imaging. Indeed, modalities such as diffusion tensor imaging, taking values in the space of symmetric positive definite matrices, or segmentation maps, taking values in the probability simplex, are recurrent in medical data. Metamorphoses aim to equip image spaces with a Riemannian structure, accounting for both the action of a group of deformations and the intensity variations of a template image. We will start this presentation by introducing the Large Deformation Diffeomorphic Metric Mapping (LDDMM) and Metamorphoses frameworks. We will then discuss the generalization of the latter model to manifold-valued images, with theoretical guarantees on the existence of solutions. We will end the presentation with numerical considerations and a few illustrating examples.

Géométrie sous-riemannienne invariante à droite en dimension infinie et applications au problème de matching d'images

Thomas Pierron

Ce travail s'ouvre sur le problème de matching d'images et le modèle LDDMM (Large deformation diffeomorphic metric mapping). Ce modèle s'applique tout particulièrement en anatomie computationnelle et imagerie médicale, notamment de par les nouvelles technologies développées récemment (IRM, technologie MERFish pour la transcriptomie, etc...). L'idée, étant données une image source et une image cible, est de trouver une deformation par difféomorphisme envoyant la source sur la cible. Ce difféomorphisme est en particulier donné par l'intégration d'un flot, dont on controle la norme pour définir des notions d'énergie et de distance. Cette approche revient alors à étudier des géométries riemanniennes (et sous-riemanniennes) invariantes à droite sur le groupe des difféomorphismes, qui se trouve être un groupe de dimension infinie. On en déduit alors les équations géodésiques et on retrouve en particulier les équations d'Euler-Arnold, qui avec certaines métriques donnent des équations de la mécanique des fluides.

Absolue continuité et feuilletages invariants

Virgile Tapiero

On peut associer à une application du plan projectif complexe une mesure de probabilité invariante, appelée mesure d'équilibre, qui reflète l'équidistribution des points périodiques répulsifs de l'application. Le taux de croissance exponentiel des valeurs singulières de la différentielle des itérées de l'application est donné par deux exposants que l'on appelle les exposants de Lyapunov de la mesure de probabilité. Lorsque ces derniers vérifient des relations particulières, on peut étudier la régularité de la mesure de probabilité et décrire le comportement géométrique de l'application. Sous une certaine hypothèse d'absolue continuité de la mesure de probabilité, je démontre dans ma thèse que l'application préserve des objets géométriques que l'on appelle des feuilletages. Un feuilletage est décrit par les courbes intégrales d'une équation différentielle holomorphe.

Tuesday 14:00-15:00

Control – e.068

On the null controllability of the heat equation with nonlinear Robin boundary conditions and M constraints on the state

Arnaud Fournier

In this paper, we consider the boundary null controllability problem of the heat equation with nonlinear Robin conditions and with M constraints on the state. This nonlinearity is of class C^1, globally Lipschitz and monotone increasing. We consider a control acting on a small part of the boundary. In the beginning, we study the linearized boundary null controllability problem with M constraints on the state by transforming it to an equivalent one with constraint on the control. With this in mind, we have established new Carleman inequalities. And finally, we use the Schauder fixed point theorem on the boundary to prove that the same result holds for the linear heat equation with nonlinear Robin boundary conditions.

On a bilinear optimal control problem for a degenerate parabolic equation with a nonlocal space term

James Larrouy

Nonlocal models are ubiquitous in all branches of science ; they express the fact that the evolution of a population at a given point in space depends not only on the surrounding density, but also on the amount of population as a whole. Nonlocal reaction terms can also describe the Darwinian evolution of the density of a structured population, or the behavior of cancer cells under the effect of therapy. In this talk, we consider a degenerate parabolic equation with a nonlocal space term (accounted by a spatial convolution kernel) involving a multiplicative control. After securing the well-posedness of the problem, we derive several properties from the control-to-state mapping. We then prove the existence of at least one optimal control for the problem under study, and establish first and second order optimality conditions leading to local uniqueness of an optimal control. Finally, we demonstrate that we are able to reach global uniqueness of optimal control by means of additional assumptions.

Numerical methods for PDEs – sc.071

Convergence acceleration of iterative sequences for reactive transport

Rawaa Awada

Numerical benchmark can be an efficient way to validate reactive transport codes. In this work, the reactive transport benchmark of GNR MoMaS is solved on its easy 1D version. We couple chemistry with transport, using the finite difference method to solve the transport equation, and
the combination of the positive continued fraction method with Anderson's acceleration method to solve the chemical equilibrium. The main advantage of this approach is to avoid forming the Jacobian matrix of the Newton Raphson method often used for solving chemical equilibriums. In addition, Anderson's speed up method is to reduce the conditional number of the matrix of the least squares problem in the implementation so that numerical stability can be guaranteed. We compare our numerical results with those obtained with the reactive transport code SPECY and we show the high efficiency of our approach.

Multi-scale Finite Element Method for incompressible flow in Perforated Domain

Loic Balazi

Simulating the flow in a multi-scale media with many obstacles, such as nuclear reactor cores, is very challenging. Indeed, to capture the finest scales of the flow, one needs to use a very fine mesh, which often leads to intractable simulations due to the lack of computational resources. To overcome this limitation, various multi-scale methods have been developed in the literature to attempt to resolve scales below the coarse mesh scale. In this contribution, we focus on the Multi-scale Finite Element Method (MsFEM).
The MsFEM uses a coarse mesh on which one defines basis functions which are no longer the classical polynomial basis functions of finite elements, but which solve fluid mechanics equations on the elements of the coarse mesh. These functions are themselves numerically approximated on a fine mesh taking into account all the geometric details, which gives the multi-scale aspect of this method.

Based on the work of (1, 2), we develop an enriched non-conforming MsFEM to solve viscous incompressible flow in heterogeneous media. Our MsFEM is in the vein of the classical non-conforming Crouzeix-Raviart finite element method with high-order weighting functions. We perform a rigorous theoretical study of our MsFEM in two and three dimensions at both continuous and discrete levels. At the numerical level, we implement the MsFEM to solve the Stokes and the Oseen problems, in two and three dimensions, in a massively parallel framework in FreeFEM (3).

The perspective of this work is to be able to solve the Navier-Stokes equations in a perforated domain at high Reynolds number using MsFEM basis functions. Furthermore, to complete the study of our MsFEM, we are investigating on an a posteriori error estimate for MsFEM.

(1) G. Jankowiak, A. Lozinski (2018). Non-Conforming Multiscale Finite Element Method for Stokes Flows in Heterogeneous Media. Part II : error estimates for periodic microstructure. arXiv:1802.04389(math).

(2) Q. Feng, G. Allaire, and P. Omnes (2022). Enriched Nonconforming Multiscale Finite Element Method for Stokes Flows in Heterogeneous Media Based on High-order Weighting Functions. Multiscale Model. Simul., 20(1):462–492.

(3) F. Hecht (2012). New development in freefem++. J. Numer. Math., 20(3-4), 251–265.

Statistics – sd.206

Procédure de test d'hypothèses composites pour l'analyse jointe de séries de probabilités critiques corrélées.

Annaïg De Walsche

L'analyse jointe de résultats de différentes expériences pour identifier des configurations complexes ou pour améliorer la puissance statistique est un objectif typique de l'intégration de données. On considère ici le cas d'une collection d'éléments i=1,..n (par exemple des gènes) dont l'effet a été testé dans différentes conditions k=1,...,K. Chaque observation i consiste donc en un vecteur de K probabilités critiques. L'objectif de l'analyse est alors d'identifier les éléments qui ont un effet dans toutes les conditions ou dans un sous-ensemble prédéfini de conditions. Les probabilités critiques doivent alors être combinées de manière flexible afin d'explorer des hypothèses complexes (appelées hypothèses composites), tout en contrôlant le taux de faux positif.

Une procédure de test d'hypothèses composites utilisant un modèle de mélange multivarié où chaque élément appartient à une classe caractérisée par une combinaison de H0 et H1 (appelé configuration) a récemment été proposée (1). La principale limite de la procédure est qu'elle repose sur une hypothèse d'indépendance conditionnelle, i.e. la loi jointe des K probabilités critiques conditionnellement à la configuration est une loi produit.

Nous proposons d'adapter le modèle pour prendre en compte la structure de dépendance conditionnelle entre les K probabilités critiques. La dépendance est modélisée dans les lois jointes conditionnelles à l'aide d'une copule. L'inférence des paramètres du modèle est réalisé via un algorithme EM. Dans l'étape (M), les paramètres de la copule sont estimés par maximum de vraisemblance implémenté par une descente de gradient réalisée par différentiation automatique. L'étape (E) est optimisée pour limiter l'empreinte mémoire de la procédure, passant de O(n x 2^K) à O(n + 2^K).

Des applications sur des données simulées ont été réalisées donnant des résultats concluants tant en termes de contrôle de faux positif et de puissance de détection qu'en terme d'efficacité de la méthode (temps de calcul et gestion de la mémoire). L'intérêt de la méthode est illustré par une application sur des données réelles d'études d'association génétique, où le nombre d'éléments (marqueurs génétiques) peut atteindre 10^6 et les nombre de conditions est de l'ordre de 10-20.
(1) T. Mary-Huard, S. Das, I. Mukhopadhyay, and S. Robin. Querying multiple sets of p-values. 2021

Nonparametric density estimation for the small jumps of Lévy processes

Taher Jalal

We consider the problem of estimating the density of the process associated with the small jumps of a pure jump Lévy process, possibly of infinite variation, from discrete observations of one trajectory. The interest of such a question relies on the observation that even when the Lévy measure is known, the density of the increments of the small jumps of the process cannot be computed. We discuss results both from low and high frequency observations. In a low frequency setting, assuming the Lévy density associated with the jumps larger than 1 in absolute value is known, a spectral estimator relying on the deconvolution structure of the problem achieves minimax parametric rates of convergence with respect to the integrated L2 loss. In a high frequency setting it is possible to remove the assumption that the Lévy measure of the large jumps is known. In that case the rate of convergence depends on the sampling scheme and on the behaviour of the Lévy measure in a neighborhood of zero. An adaptive penalized procedure is also proposed to select the cutoff parameter.

Tuesday 15:30-17:00

Theoretical analysis and numberics of PDEs – e.068

Traveling waves for a quasilinear Gross-Pitaevskii equation

Erwan Le Quiniou

We consider a modified *quasilinear* version of the Gross-Pitaevskii equation. Intuitively this is a nonlinear Schrödinger equation where the terms involving spatial dervatives depend on the unknown function. We also recquire the solutions to satisfy *nonzero* conditions at infinity.
In this setting we classify particular solution of these PDEs—the traveling waves of finite energy—in terms of two parameters: the propagation speed of the wave and a real number measuring the importance of the quasilinear terms. Furthermore, we prove that some of these waves can be obtained by minimization of the energy at fixed momentum, and that they are orbitably stable.

The aim of this talk is to presents our results. We will discuss the properties of the admissible travelling wave profiles to conclude on a sketch of the proof for the orbital stability of some solutions.
If i'm not selected to present during this event I would like to prepare a poster instead.

A landscape evolution model: mathematical analysis and numerical scheme

Julie Binard

In this talk I will present a system of partial differential equations modeling the evolution of a landscape, under the effects of erosion caused by the shear strees of water flow, and sedimentation. The system studied is composed by three evolution equations on the elevation of the ground surface, the fluid height, and the concentration of sediment in the fluid layer. We will see that the system is well-posed locally in time under some assumptions on the data and the parameters. Then, we will focus on pattern formation for this model, when the initial topography is an inclined plane and the bottom surface is weakly eroded. These patterns, which are rills and gullies, are the starting point of the formation of rivers and valleys in landscapes. To identify instability mechanisms that could explain the formation of patterns, we linearise System (1) around a constant state and we study the conditions of spectral instability and the nature of the instabilities. Finally, we will illustrate the appearance of patterns for the nonlinear system with numerical simulations.

Electromagnetic waves propagation in thin heterogenous coaxial cables. Comparaison between 3D and 1D models.

Akram Beni Hamad

This work deals with wave propagation into a coaxial cable, which can be modelled by the 3D Maxwell equations or 1D simplified models. The usual one, called the telegrapher's model, is a 1D wave equation on the electrical voltage and current. We derived a more accurate model from the Maxwell equations that takes into account dispersive effects. These two models aim to be a good approximation of the 3D electromagnetic fields in the case where the thinness of the cable is small. We perform some numerical simulations of the 3D Maxwell equations and of the 1D simplified models in order to validate the usual model and the new one. Moreover, we show that while the usual telegrapher model is of order one with respect to the thinness of the cable, the dispersive 1D model is of order two.

Optimisation – sc.071

Properties of Discrete Sliced Wasserstein Losses

Eloi Tanguy

The Sliced Wasserstein (SW) distance has become a popular alternative to the Wasserstein distance for comparing probability measures. Widespread applications include image processing, domain adaptation and generative modelling, where it is common to optimise some parameters in order to minimise SW, which serves as a loss function between discrete probability measures (since measures admitting densities are numerically unattainable).
All these optimisation problems bear the same sub-problem, which is minimising the the SW distance between two uniform discrete measures with the same amount of points, as a function of the support of one of the measures. We investigate the regularity and optimisation properties of this energy E, as well as its Monte-Carlo approximation E_p (estimating the expectation in SW using only p samples) and show convergence results on the critical points of E_p to those of E, as well as an almost-sure uniform convergence.
Furthermore, we show that in a certain sense, Stochastic Gradient Descent (SGD) methods minimising E and E_p converge towards (Clarke) critical points of these energies. Finally, we leverage these insights in order to establish the convergence of SGD methods optimising the SW loss for generation tasks.

Shape optimization of harmonic helicity in toroidal domains

Robin Roussel

In this talk, we introduce the notion of harmonic helicity of a toroidal domain, and discuss its shape optimization both from the theoretical and numerical point of view. Given a toroidal domain, we consider its associated harmonic field. The latter is the magnetic field obtained uniquely up to normalization when imposing zero normal trace and zero electrical current inside the domain. We then study the helicity of this field, which is a quantity of interest in magneto-hydrodynamics corresponding to the L^2 product of the field with its image by the Biot-Savart operator. To do so, we begin by discussing the appropriate functional framework and an equivalent PDE characterization. We then discuss shape optimization, and we identify the shape gradient of the harmonic helicity. Finally, we discuss a numerical scheme to compute harmonic helicity and its gradient using finite elements exterior calculus, and present some simulations for the shape optimization problem.

Tree-Based Diffusion Schrödinger Bridge with Applications to Wasserstein Barycenters

Maxence Noble

Multi-marginal Optimal Transport (mOT), a generalization of OT, aims at minimizing the integral of a cost function with respect to a distribution with some prescribed marginals. In this paper, we consider an entropic version of mOT with a tree-structured quadratic cost, i.e., a function that can be written as a sum of pairwise cost functions between the nodes of a tree. To address this problem, we develop Tree-based Diffusion Schrödinger Bridge (TreeDSB), an extension of the Diffusion Schrödinger Bridge (DSB) algorithm. TreeDSB corresponds to a dynamic and continuous state-space counterpart of the multimarginal Sinkhorn algorithm. A notable use case of our methodology is to compute Wasserstein barycenters which can be recast as the solution of a mOT problem on a star-shaped tree. We demonstrate that our methodology can be applied in high-dimensional settings such as image interpolation and Bayesian fusion.

Mathematics in health and biology – sd.206

Modélisation de la dynamique des membranes à courbure variable sous l'effet des fluctuations thermiques.

Mohammed Adel Djibaoui

Du fait de leur organisation et des fonctions multiples qui leur sont conférées par leur structure complexe, les membranes cellulaires présentent un intérêt à la fois fondamental et applicatif mais leur approche nécessite souvent une approche interdisciplinaire combinant outils mathématiques et physiques et connaissances biologiques. Reflet de cette complexité, l'élucidation de la relation entre leurs structures et leurs fonctions multiples se heurte à des obstacles majeurs. D'origine structurale et fonctionnelle, la complexité des membranes biologiques résulte d'un vaste ensemble d'interactions moléculaires s'exprimant dans les processus biologiques mais aussi de l'exploitation de couplages naturels entre divers mécanismes physico-chimiques. De nombreux modèles relevant de la Biophysique visent une description mécanique et thermodynamique de ces membranes s'appuyant sur la fameuse fonctionnelle (énergie) de Canham-Helfrisch (3, 2), prenant en compte la forme, c'est-à-dire la géométrie et la topologie des membranes et permettant la détermination des configurations d'équilibre des membranes. Du point de vue mécanique, ces membranes présentent les caractéristiques combinées de solides et de fluides (écoulement interne à la membrane comme évoqué dans le vieux modèle de la mosaïque fluide (5)). L'étude présentée ici concerne un modèle mécanique simple de membranes cellulaires planes ou courbées et permettant de décrire les interactions induites entre protéines incluses dans celle-ci (1). Basé sur une extension simple de la fonctionnelle de Helfrisch, le modèle décrit d'abord la dynamique déterministe des déplacements normaux de membranes élastiques homogènes et isotropes. Le spectre des excitations de basse énergie (ondes de courbure) est discuté dans le cas de membranes de haute symétrie telles les vésicules sphériques ainsi que la forme générale de la compliance de celles-ci. Dans une seconde étape, la théorie générale des fluctuations thermiques (4) de la membrane est présentée ainsi que ses effets sur les interactions résiduelles entre protéines incluses, de nature entropique. La forme du potentiel d'interaction correspondant est calculée numériquement dans le cas de membranes planes. Le rôle potentiel de ces interactions sur l'organisation des protéines au sein de la membrane est discuté. Leurs réorganisations au sein de la membrane pourraient être approchées efficacement comme transitions de phases de la membrane 'habillée' par une analogie intéressante de notre modèle avec le gaz de Coulomb 2D.

Individual based infection models on (not so) dense large random networks

Aurélien Velleret

Starting from a stochastic individual based description of an SIS epidemic spreading on a random network, we will look at the dynamics as the size of the network tends to infinity. We recover in the limit an infinite-dimensional integro-differential equation studied by Delmas, Dronnier and Zitt (2022) for an SIS epidemic propagating on a graphon. Our work covers the case of dense and sparse graphs, when the number of edges is of order 0(n^a) with a $\in$ (1, 2) (the case of very sparse graphs with a = 1 and boundedness properties in the number of neighbors is of different nature). As a crucial ingredient for our limit theorem, I will mention the essential properties of a coupling between the process of interest and an epidemic spreading with a modified infectivity on the complete graph. The proof naturally leads to a new summary statistic to quantify the convergence. I will thus present in typical examples the behavior of this average over the infection rate per link. Joint work with J-F. Delmas, P. Frasca, F. Garin, V.C. Tran and P-A. Zitt

Nonparametric estimator of the distribution of fitness effects of new mutations

Guillaume Garnier

Mutations are an essential mechanism that plays a key role in the history of life; in particular, they explain the appearance of new hereditary traits within a population.
These new traits can modify the selective or fitness value of an individual. In evolutionary biology, biologists are interested in distribution of fitness effects (DFE) of new mutations since it is a key element to understanding the evolutionary trajectory of a population.

In this talk, we present a probabilistic model based on compound Poisson processes that describes the evolution of the fitness of a cell line over the time . We propose an nonparametric estimator of the DFE based on the noisy observation of a i.i.d. sample of cell line over discrete time. In particular, it is not possible to observe fitness jumps related to the occurrence of a new mutation.
We use a Fourier approach to construct this estimator, to provide risk bounds and an adaptive procedure.

Workshops, Tuesday 17:15-18:45

Stochastic differential equations – sc.071

PDE techniques in numerical analysis of SDEs

Ludovic Goudenège

Dans ce workshop nous étudierons la convergence d'un schéma numérique de type Euler pour la résolution d'une équation différentielle stochastique. Les techniques de la démonstration feront appel à des équations aux dérivées partielles de Kolmogorov associées au générateur infinitésimal de l'équation différentielle stochastique. On s'appuiera également sur des simulations numériques pour visualiser les vitesses de convergence.

Statistics and Machine learning – e.068

Some Mathematical Results and Open Questions on (deep) Neural-Networks

Christophe Giraud

Machine learning algorithms based on Neural-Networks (NN) achieve some stunning empirical performances, which seemed completely out of reach 10 years ago. While theses algorithms have been largely developed on a try-and-error basis on data challenges, mathematics have an important role to play for understanding their behavior, and for producing certificates on their predictions. Classical machine learning theory fails to grasp the main features of NN, and building a suitable theory for understanding NN is a major challenge that mobilizes many researchers worldwide. In this survey, we will discuss a biased selection of recent results on neural networks, and highlight some important open questions.

Analysis of PDEs – sd.206

TBA

Anne-Laure Dalibard

TBA

Wednesday 9:30-11:00

Numerical methods for PDEs – e.068

Sensitivity analysis for incompressible Navier-Stokes equations

Nathalie Nouaime

The Navier-Stokes equations are a system of PDEs which are nonlinear and involve second-order derivatives of the fluid velocity, making them difficult to solve analytically. Our present study focuses on sensitivity analysis for the Navier-Stokes equations, with an emphasis on the stability estimate of the discretised first-order sensitivity of the Navier-Stokes equations. Sensitivity analysis studies how changes in the input of a model affect the output. This task can be performed in many different ways, depending on the nature of the model considered. The present study focuses on sensitivity analysis of incompressible Navier–Stokes equations using the polynomial chaos method. First, the firstorder sensitivity of the Navier-Stokes equations is defined. A finite element-volume numerical scheme for the Navier-Stokes equations is proposed. This discretisation is integrated into the open-source industrial code TrioCFD promoted by the CEA. Second, the finite element-volume discretisation is extended to the first-order sensitivity Navier-Stokes equations; the most significant point is the discretisation of the non linear term. Finally, a stability estimate for the continuous and discrete Navier-Stokes equations is established.

Schémas implicites semi-Lagrangiens pour la dynamique des gaz compressibles

Alexiane Plessier

Ce travail s'inscrit dans le cadre plus général de la dynamique des fluides. Ce qui nous intéresse particulièrement dans ce contexte est l'interaction fluide-structure et plus précisément le cas de structures internes fines en formalisme semi-Lagrangien.
Pour approcher les équations traduisant le mouvement des fluides, on utilise traditionnellement des schémas explicites qui pour être stables sont sujets à une condition CFL. Dans le cas qui nous intéresse, l'épaisseur de la structure peut-être très fine et la vitesse du son très grande. Il est donc nécessaire de prendre un pas de temps très petit et par conséquent, il est difficile d'obtenir de bons résultats numériques à faibles coûts.

Pour remédier à ce problème, l'idée est d'utiliser localement des schémas implicites. Néanmoins, des difficultés techniques majeures apparaissent, notamment celle de montrer que le schéma est bien défini (existence et unicité d'une solution au temps suivant). Nous proposons un schéma implicite non linéaire pour la partie hydrodynamique qui résout les équations d'Euler compressibles multi-D écrites en formalisme semi-Lagrangien.

Ce schéma implicite non linéaire est basé sur une méthode de prédiction-correction. La phase de prédiction résout les équations d'Euler isentropiques et la phase de correction correspond à la discrétisation des équations d'Euler avec conservation de l'énergie totale. Un théorème d'existence et d'unicité d'une solution au schéma implicite sera évoqué. Une attention spécifique sera portée à la définition d'un couplage explicite-implicite utilisé lors de simulations bi-fluides.
Quelques résultats numériques 1D et 2D seront présentés dans le but de montrer certaines propriétés du schéma ainsi que d'analyser la sensibilité du schéma aux différents paramétrages (pas de temps, maillage, solveur linéaire, solveur Volumes Finis).

Modeling of a 1D gas flow under a Low Mach regime in a Thermosiphon

Giuseppe Parasiliti Rantone

Natural gas is one of the most worldwide spread energy resources, and optimizing its transportation and distribution is fundamental. The main issue during transport is energy and pressure loss caused by the friction of the gas against the walls of the pipes and the fact that the gas does not constitute an isolated system, which implies that heat exchange with its surroundings is possible. Since the velocity inside the transportation pipes is often small compared to the speed of sound, it is possible to make asymptotic approximations in modeling such gas flows. We preferred to put our efforts into studying a "Low Mach" model rather than the more widespread "Boussinesq approximation" to gain accuracy. One of the most archetypal flow problems in a pipe with heat exchange is the "thermosiphon". It is a closed pipe of length 4L where the gas is confined at some mean pressure in the gravity field and flows in a loop (here, a square) from one canal to another. The one at temperature Tf cools the gas, so it is denser and falls, and the one at temperature Tc heats it, so it is less dense and moves up. The inclination of the pipes *theta* depends on the geometry: in the heated pipe, it is $\pi/2$, in the cooled one, it is $-\pi/2$ and in the others, 0. In the adiabatic parts, gas experiences no heat transfer with its surroundings. We considered an averaged version across the section of a pipe of compressible Navier-Stokes equations for ideal gases, and we derived an asymptotic model by taking the Mach number small. We obtained a quasi-compressible one-dimensional model, with the velocity divergence small but not null (due to heating); as a result, we managed to decouple the pressure into two terms: dynamical pressure plays a fundamental role in the conservation of momentum, and thermodynamical pressure is present in the conservation of energy. We found the analytical solutions to the steady laminar version of the model; these solutions are crucial for validating the simulations performed. As the next step, we designed a robust and efficient numerical method to simulate the problem. We proposed a discretization of temperature through the method of characteristics, while for the other variables, we used both implicit and explicit finite-difference schemes. We coupled the computation of dynamic pressure and velocity through the projection technique. We faced the issues of periodic boundary conditions and the discontinuous gravity term with Dirac distributions as derivatives at the corners. We found another equation to consider the evolution in time of the thermodynamical pressure. The results prove our algorithm is accurate, robust, and efficient.

PDEs, probability and stochastic control – sc.071

Controlled superprocesses and HJB equation in the space of finite measures

Antonio Ocello

This talk introduces the formalism required to analyze a certain class of stochastic control problems that involve a super diffusion as the underlying controlled system. To establish the existence of these processes, we show that they are weak scaling limits of controlled branching processes. By proving their uniqueness in law, we can establish a dynamic programming principle for our stochastic control problem. This lays the groundwork for a PDE characterization of the associated value function, which is based on the concept of derivations in the space of finite positive measures. We also establish a verification theorem. To illustrate this approach, we focus on an exponential-type value function and show how a regular solution to a finite-dimensional HJB equation can be used to construct a smooth solution to the HJB equation in the space of finite measures.

Scattered wavefield in the stochastic homogenization regime

Quentin Goepfert

We aim at providing a mathematical framework for the scattering of ultrasound waves by a bounded object a random composite microstructure embedded in an unbounded homogeneous background. The problem considered is modeled by a divergence form scalar Helmholtz equation with discontinuous rapidly oscillating (at some scale ϵ much smaller than the wavelength) stochastic coefficients. We perform an asymptotic analysis with respect to ϵ using stochastic homogenization techniques. We obtain a high order approximation, quantify the error with the reference model and perform numerical simulation to illustrate the convergence.

On the optimal rate for the convergence problem in mean-field control

Samuel Daudin

We are interested in control problem in the space of proabability measures over the euclidean space, which arise as limit of control problems involving a large number of interacting particles driven by stochastic differential equations. In this joint work with François Delarue and Joe Jackson we obtain optimal rates of convergence for the value functions.
Our analysis covers cases where the solutions to the limiting problem may not be unique nor stable. Equivalently the value function of the limiting problem might not be differentiable on the entire space. Our main result is then to derive sharp rates of convergence in two distinct regimes. When the data is sufficiently regular, we obtain rates proportional to $N^{-1/2}$, with N being the number of particles. When the data is merely Lipschitz and semi-concave with respect to the first Wasserstein distance, we obtain rates proportional to $N^{-2/(3d+6)}$. Noticeably, the exponent $2/(3d+6)$ is close to $1/d$, which is the optimal rate of convergence for uncontrolled particle systems driven by data with a similar regularity. The key argument in our approach consists in mollifying the value function of the limiting problem in order to produce functions that are almost classical sub-solutions to the limiting Hamilton-Jacobi equation (which is a PDE set on the space of probability measures). These sub-solutions can be projected onto finite dimensional spaces and then compared with the value functions associated with the particle systems. In the end, this comparison is used to prove the most demanding bound in the estimates. The key challenge therein is thus to exhibit an appropriate form of mollification.
We do so by employing sup-convolution within a convenient functional Hilbert space. To make the whole easier, we limit ourselves to the periodic setting. We also provide some examples to show that our results are sharp up to some extent.
This is based on the preprint arXiv:2305.08423

Machine learning, neural networks – sd.206

Getting large language models better at mathematics

Fabian Glöckle

Large language models such as ChatGPT have become increasingly versatile general purpose information retrieval and information processing tools, yet they frequently fail at basic logical reasoning problems and suffer from hallucinations, which limits their usefulness for mathematical research. To ground them in mathematical semantics, computer-assisted proof languages like Lean, Isabelle and Coq can be used as environments that provide feedback on logical correctness. However, these suffer from training data scarcity relative to the data-hungry transformer language model architectures used. Recent works therefore bootstrap proof data from a combinatorial search procedure using neural networks like AlphaGo which is used to incrementally retrain the language model. With the advent of large language models, new avenues for adapting language models for mathematics have opened up. In my talk, I will argue that logical soundness might be best understood as a problem of adapting the base capabilities of the model to the task of mathematics, and that in turn techniques from alignment research like reinforcement learning from AI feedback could take the place of combinatorial search in the current state of the art.

On the Generalization Capacities of Neural Controlled Differential Equations

Linus Bleistein

Time series are ubiquitous in many domains such as finance, agriculture, economics and healthcare. A common set of tasks consists in predicting an outcome, such as a scalar or a label, from a time-evolving set of features. This problem has been addressed with a great variety of methods, ranging from auto-regressive models, such as VAR, to deep learning models (RNN, LSTM, etc.). It has also been thoroughly studied through the lenses of stochastic processes, Gaussian processes and many more. Both in practice and theory, most methods and theoretical setups often only tackle the case of regularly sampled time series. In this setup, a time series is seen as a collection of datapoints for which the time step between two observations is constant. In real-world scenarios, however, time series are often irregularly sampled. This is often the case when data collection is difficult or expensive. From a modelling point of view, it is reasonnable to assume that the outcome does not depend on the time series itself, but on a continuous path the time series is sampled from. In agriculture, for instance, crop growth and yield is not determined by the punctual measurements of soil fertility, but by its continuous value through time. Learning with discretized data instead of continuous data therefore introduces a bias. It is of great importance to better understand this bias degrades the performance of learning algorithms. We focus on the particular case of Neural Controlled Differential Equations, or NCDE (Kidger, Morrill, et al. 2020), which are a recent state-of-the-art algorithm adressing the challenges of learning from irregular time series through controlled differential equations (CDE). They can be seen as extension of Neural ODE (Chen, et al. 2018) to sequential data. Informally, these models embed an irregular time series by using it as the driving signal of a CDE with a neural vector field, which is then optimized for the task at hand. This encompasses time series classification, regression, or forecasting. We study how irregular sampling affects the generalization capacities of NCDEs. We first design a generative model, which links the unobserved continuous path to the outcome through an unknown differential equation - this could be any differential equation model commonly used in biology or physics. We then obtain the covering number and Rademacher complexity of NCDE through recent arguments building on the Lipschitz continuity of deep neural networks. This allows us to obtain our first crucial result, which is a generalization bound for NCDEs. This result upper bounds the expected loss made with the predictor obtained by empirical risk minimization by the training loss and a complexity-dependant term. We then leverage the continuity of the flow of CDEs to precisely quantify the discretization bias. This result includes the convergence speed of this bias towards zero as the sampling of the time series gets finer. Finally, using the same technique, we are able to show that the approximation bias (which comes from the fact that we are approximating a generic differential equation by a neural differential equation) directly relates to the approximation of the vector field of the differential equation. This is a striking result, since one would excpect that this bias depends of the whole flow of the generative model. This is joint work with A. Guilloux (Inria).

Modelling of Highly oscillatory phenomenon by neural networks

Maxime Bouchereau

Ordinary differential equations are widely used to model various phenomena. However, these equations are often too complex to be analytically solved. Therefore, numerical methods are used to approximate solutions of an ODE, but many simple methods are not accurate for small computational time.
With Machine Learning, we perform existing numerical methods, by using the modified field theory, which consists in approximate modified field with neural networks, and them, efficiently solve the ODE.
The case of autonomous ODE's is first studied. Then, highly oscillatory ODE's are studied, but they are more complex to solve and require tools such that Averaging theory or more complicated numerical methods: Uniformly Accurate methods.

Panel discussion – Wednesday 11:30-12:30 – Théâtre e.070

Career opportunities in research

Maria Paula Gomez Aparicio, Flore Nabet, Stéphane Nonnenmacher, Guillaume Pierrot, Irène Vignon-Clémentel.
Chair : Angèle Niclas

Plenary talk – Wednesday 14:00-15:00 – Théâtre e.070

Evolution de la diversité génétique d'une population ayant une structure spatiale

Amandine Veber

La structure spatiale d'une population influence la manière dont les gènes sont transmis d'une génération à l'autre et, par conséquent, la manière dont la diversité génétique de la population évolue au cours du temps. Pour modéliser cette dynamique dans un espace continu, un processus stochastique appelé "processus Lambda-Fleming-Viot spatial" a été introduit en 2008 par Alison Etheridge (Oxford Univ.) et Nick Barton (IST Austria), et a été développé dans diverses directions depuis. Nous présenterons ce modèle, les processus "généalogiques" correspondants et nous discuterons quelques résultats obtenus grâce à cette approche.